



Liu, M., Agarwal, J., & Blockley, D. I. (2016). Vulnerability of road networks. *Civil Engineering and Environmental Systems*, 33(2), 147-175. <https://doi.org/10.1080/10286608.2016.1148142>

Peer reviewed version

Link to published version (if available):
[10.1080/10286608.2016.1148142](https://doi.org/10.1080/10286608.2016.1148142)

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PDF-document

This is an Accepted Manuscript of an article published by Taylor & Francis in *Civil Engineering and Environmental Systems* on 22/03/2016, available online:
<http://www.tandfonline.com/doi/full/10.1080/10286608.2016.1148142>

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Vulnerability of Road Networks

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Abstract

Current evaluations of the vulnerability of a road network tend to focus on the probability of damage and the change of traffic demand. The forecasting of low-probability but high-consequence events is a major difficulty. In this paper, a new theory, using a systems-thinking approach, for examining the vulnerability of the form of the network is presented. Our purpose is not to simulate traffic flow but to identify high consequence scenarios that may arise from vulnerable weaknesses in the form of the network. Such scenarios are independent of models of traffic demand or the source of the damage and can subsequently be combined with specific demands to assess risk. A hierarchical model with clusters of road circuits formed at various levels of granularity of a road network is developed for use in a search process. Only free uncongested flow is considered. A search algorithm for finding vulnerable failure scenarios is described. A vulnerability index is proposed as a measure of the disproportionateness of the consequences of a series of events within a failure scenario in relation to the damage causing those events. The theory is illustrated with two examples.

Keywords: vulnerability; road network; accessibility; topological analysis; low probability high consequence failure.

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Keywords: vulnerability; road network; accessibility; topological analysis; low probability high consequence failure.

1. Introduction

Many road networks have evolved in response to societal needs rather than through an overall grand design plan. It is not uncommon for such networks to display a reduced level of accessibility or travel time due to vehicle breakdowns, traffic accidents, infrastructure maintenance works, severe weather and other natural causes. High profile events such as the 1995 earthquake in Kobe, Japan, the attacks on the World Trade Centre in New York city in 2001, the 2010 volcanic ash cloud in Iceland, floods in Columbia in 2011 and Canada in 2013 have resulted in increased levels of research on the effects of disruptions on transport networks. Most of the current research is based on concepts of topology, accessibility, reliability, and vulnerability (Berdica 2002; Chen *et al* 2002; Litman 2012) which are briefly reviewed in the next section. These concepts are equally important for other infrastructure networks such as water supply (Jowitt and Xu 1993; Pinto *et al* 2010), electricity grids (Holmgren, 2006) and communication networks (Neumayer *et al* 2011).

In this paper, a new theory, using a systems-thinking approach, for examining the vulnerability of the form of a road network is presented. The purpose is to identify high consequence scenarios that may arise from weaknesses. The weaknesses may be independent of models of traffic demand or the nature of damage but not of the effort to cause that damage. In further work, not included here, these scenarios could then be combined with specific traffic demands to assess risks.

A road system is modelled as a network of nodes or vertices on a graph representing road junctions connected by links or edges as road sections. The new theory draws from graph, road traffic and structural vulnerability theories (Wu *et al* 1993) to define new measures and build a hierarchical model of different levels of granularity of a road network as a basis for a search process for failure scenarios. In Section 2, the literature on quality assessment of transport networks is reviewed briefly and the concept of vulnerability is discussed. Sections 3 and 4 present the basis for the theoretical development of the proposed method, including a new measure of the quality of a network. This is used in Section 5 to cluster the parts of a network thus leading to a hierarchical representation of the network. The algorithm for the unzipping of a hierarchical model is presented in Section 6. Section 7 contains two illustrative examples.

2. Measures of quality of road networks

2.1. Topological measures

Road networks are usually complex systems which can be represented as graphs with a large number of connected vertices and links. Topological features of the complex networks such as vertex degree, edge betweenness and network diameter are known to have a strong impact in assessing the physical properties of the networks such as robustness or vulnerability (Barrat *et al* 2004). The vertex degree is the number of links joining a vertex and is used in other measures such as (a) clustering efficiency which reflects the tendency of vertices to be clustered (Barthelemy 2011), (b) eigenvector centrality which is proportional to the sum of the degree of the neighbours of a vertex (Newman 2004) and (c) betweenness centrality which indicates the importance of a vertex for the flow between a pair of vertices (Barthelemy 2011). Network diameter is the average length of the shortest paths between any two vertices. The stability of a network (i.e. the conditions under which it becomes inefficient and cannot transmit flow within acceptable time period) has been studied by systematically removing a critical percentage of links (Lopez *et al* 2007).

Most of these measures do not include how well the vertices and links transmit flows through other aspects of the form of the network such as the lengths and capacities of links.

2.2. Accessibility

Accessibility is the ease with which road users, goods and services reach their destinations. Two parameters are usually included: (a) the attractiveness of the destination zone and (b) the cost of the trips to this zone. Hansen (1959) proposed measuring the attractiveness of this zone using the size of the activities or the number of stores and people and the costs of travel time or distance from other zones. Dalvi and Martin (1976) considered the importance of different types of attractiveness. Sohn

(2006) used the shortest distance and traffic volume to construct an accessibility index.

2.3. Reliability

The reliability of a road network is a measure of the stability of the quality of service offered to its users (Bell and Cassir 2000). It may describe a connection between vertices, referred to as ‘connectivity reliability’ (Sakakibara *et al* 2004). Alternatively it may describe the success of road users travelling along a path between a pair of vertices in an expected travel time, referred to as ‘travel time reliability’ (Nicholson and Du 1997). The term has also been used to represent the probability of network capacity being greater than or equal to a required demand level. This is referred to as a ‘capacity reliability’, when link capacity is subject to random variations due to user routing choices (Chen *et al* 2002).

2.4. Vulnerability

The Oxford English Dictionary defines vulnerability as susceptibility to damage. Berdica (2002) related vulnerability to incidents that can result in considerable reductions in the serviceability of transport networks. Results of incidents have been measured by various indices, usually consisting of the topological characteristics of network elements or traffic flow or both (Dall’Asta *et al* 2006). For example, a node or link is vulnerable if loss (or substantial degradation) of a small number of links significantly diminishes the accessibility, as measured by a standard index of accessibility (Lieras-Echeverri and Sanchez-Silva 2001; Taylor 2008). Other measures used have included reliability (Immers *et al* 2004), travel time (Qiang and Nagurney 2008; Jenelius 2009), traffic volume (Tampère *et al* 2007) and risk (Tampère *et al* 2007). Jenelius (2009) observed that a relatively small incident, if it happens in an unfortunate (critical) place and time, can cause major damage or even the failure of the whole system by chain reaction. Haimes (2006) related vulnerability to the inherent states of a system that can be exploited to adversely affect the system.

In this paper, a network is said to be vulnerable if damage to a small part of a road network results in the failure of a significant part or whole of it. In other words, vulnerability exists if a small damage causes disproportionate consequences (Blockley 2010).

Many different network metrics (e.g. Grubestic *et al* 2006) have been used for assessing the vulnerability of general networks. Typically, evaluations are based on graph-theoretic measures (Section 2.1) and do not consider actual flows, distances or capacities. Simulations of actual demands and capacities cannot be relied upon as demands or capacities change. The method presented in this paper addresses these limitations. The consequences of damage are evaluated by a change in the performance measure called ‘well-formedness’ (Section 4) which is related to the form of a network (including lengths, capacities etc.) but is independent of the traffic demand. Vulnerability combined with the likelihood or the number of occurrence of damage

events leads to risk.

A robust road network should be able to function at an acceptable service level when subjected to arbitrary uncertain conditions. A resilient network is not only robust but it has the ability to quickly recover from an adverse state. A sufficient condition for a lack of robustness and resilience is that a network is vulnerable.

3. Modelling of road networks

3.1. *Graph model*

A road network can be modelled by a graph where the road locations and sections are represented by vertices and links. However different levels of granularity and appropriate choices of vertices and links will be needed as appropriate for a given application. Granularity means having many distinct parts. Levels of granularity means having more than one layer of definition of a system with different numbers of defined distinct parts. Only uncongested networks will be considered in this work. Figure 1 shows a ‘high level’ model of the Motorways and ‘A’ roads in England in which the vertices are some major cities and towns as well as major junctions or interchanges. The links connecting the vertices may not be single direct roads but rather road pathways. The distance between any two vertices is then the actual length of one of the paths (for example, the actual shortest path) between them. Traffic originates and ends at vertices.

The graph model can be interpreted in different ways, depending upon the context. For example, if trips between London, Bristol, Brighton, Birmingham, Leeds, and Newcastle upon Tyne are to be analysed, a more appropriate graph model may be that shown in Figure 2(a). Here only those cities and towns are considered as origins/destinations. One or more of them may be chosen as ‘reference nodes’ for a particular purpose such as an analysis of the vulnerability of the road networks connecting one city with several others. The other vertices are considered as junctions and referred to as ‘internal nodes’ or simply the nodes. The sizes of the reference nodes denote the scale of the cities such as population, economic importance or the geographical size. Figure 2(b) is a higher granularity level (coarser) model of Figure 2(a) where the internal nodes and links have been removed leaving only direct links.

[Figure 1 near here]

[Figure 2 near here]

3.2. *Road trips and circuits*

A road trip is usually defined as the movement of traffic along a path of links between various nodes from origin to destination (which may or may not also be the origin). In order to develop a theory for the vulnerability of road networks the elemental concept

used here is that of a road circuit. A road circuit is defined as a closed loop which can be traversed without visiting any other node more than once. Any path (and hence any road trip) between any two nodes is contained in a circuit or a combination of circuits. However, the focus is not on individual journeys rather on the state of network as a whole.

A basic road circuit, or a Hamiltonian Cycle in graph theory terms, is one in which the vertices are not connected by links which are not part of the circuit. For example, in Figure 3 circuits $x_1-x_2-x_3$ and $x_1-x_3-x_4$ are basic circuits but $x_1-x_2-x_3-x_4$ is not basic because link l_5 is not part of the circuit.

[Figure 3 near here]

A set of basic circuits in a road network can be obtained from the corresponding graph. The number of basic circuits (two in Figure 3) is equal to the cyclomatic number of the graph (Christofides 1975). The basic circuits will later be used to form clusters.

Traffic potential is the desire, need or demand to travel expressed as a number of standard passenger car units (pcu) per hour. Traffic flow is the flow of vehicles per hour per lane. Our purpose however is not to model or simulate individual journeys from source to sink but rather to examine the overall stability of a traffic network and its vulnerability to damage. For this we examine the overall connectivity of population centres as both sources and sinks of traffic. When there is more than one pathway in either direction between a source and a sink then a circuit including that source and sink must exist. Multiple interconnected circuits form a hierarchy of connected clusters of circuit (Section 5). The hierarchy enables us to identify multiple paths between sources and sinks at various levels of granularity.

The form of a circuit is crucial to the way in which it resists any arbitrary demand by redirecting flow along alternative pathways. Damage is any disruption to the flow along a path. Arbitrary traffic demands, even if not already present, may arise due to unpredictable damage or changes in land use. The flow along a road path may be disrupted by road works or an accident.

4. Measures of well-formedness

Well-formedness is a measure of the quality of the form of a network. The nature of well-formedness will vary for different physical systems but there are underlying similarities. We use an analogy between a structure and a road network in order to examine the possible effects of damage. The rings of structural vulnerability theory (Wu *et al* 1993; Agarwal *et al* 2001) are circuits forming load paths whilst a road circuit provides traffic paths. The well-formedness measure successfully used for static (not dynamic) structures consists of the principal stiffness coefficients of the members and joints. It gives the capability of a structure to resist loading from any arbitrary direction.

For traffic it is postulated that a new concept of ‘continuance’ is a basis for the development of a measure of the well-formedness of a road network with uncongested flow.

The well-formedness of a road network, similar to structural well-formedness, should:

- increase with an increasing capacity of traffic flows,
- increase with measures of the quality of nodes (vertices or traffic junctions),
- increase with a higher connectivity within the network and
- be independent of the chosen coordinate system

4.1. Continuance of a link

Continuance is a new concept developed to represent the continuation, duration or maintenance of travel time with different degrees of saturation of uncongested traffic flow. It is an inductance as a lag through a storage of flow but does not include delays through congestion or inertance (which is related to mass and acceleration of fluid flow). It is a lag that varies with density of flow through the varying space gap between vehicles as drivers make judgements about safe stopping distances.

A measure of continuance was developed through an analogy with Young’s modulus in structural mechanics. Traffic flow (q) is assumed as analogous to stress (σ) and traffic strain (μ) to mechanical strain (ϵ). A traffic modulus (K) is thus:

$$K = \frac{q}{\mu} \quad (1)$$

Traffic strain is the ratio of the (difference between the actual travel time t_a and the free uninterrupted flow travel time t_f) to the (free uninterrupted flow travel time t_f) and dimensionless but expressed in time (rather than in length as in mechanics). Free uninterrupted flow travel time is the travel time on a road section when the traffic flows uninterruptedly and the environmental conditions such as weather and road conditions are not problematic. For a road section of length L , Equation (1) can be rewritten as:

$$K = \frac{q}{\mu} = q \cdot \frac{t_f}{(t_a - t_f)} \quad (2)$$

and also

$$q = \frac{K}{t_f} \cdot t_a - K \quad (3)$$

Relationships between flow, density and speed have been modelled by different functions (see e.g. Kerner 2009). A modified form of the Akçelik curve (Akçelik 2006)

for uncongested flow, i.e. larger uncongested flow is a linear function of travel time as shown in Figure 4, is used here. t_c represents capacity flow travel time and is always greater than t_f .

[Figure 4 near here]

It is also assumed that when $q = 0$, t_c is equal to t_f . Uncongested flow will be assumed for the vulnerability analysis presented here but it is important to note that when $t_a = t_c$, traffic is flowing at its capacity i.e. $q = Q$. Any further increase in traffic concentration (i.e. vehicles per unit distance) leads to a drop in speed and the actual travel time could become very high and flow rates very low (labelled as congested flow in Figure 4). We consider congestion as a degradation in the quality of the network and hence partial damage to one or more links. Travel time can also increase when there is a wide moving jam flow. Kerner (2009) notes that such flow maintains the mean speed of the downstream front of the jam as the jam propagates upstream and vehicles accelerate at the downstream jam front from low speeds states inside the jam to higher speeds as they leave the jam. We consider this as a special case of congested flow and is not considered for the vulnerability analysis.

In Figure 4 for uncongested traffic flow with $q < Q$, the relationship between the flow and actual travel time is:

$$\frac{q}{Q} = \frac{1}{t_c - t_f} \cdot t_a - \frac{t_f}{t_c - t_f} \quad (4)$$

where q/Q is the degree of saturation of uncongested flow. Also

$$q = \frac{Q}{(t_c - t_f)L} \cdot (t_a - t_f) \cdot L \quad \text{for } 0 < q < Q \quad (5)$$

The continuance of a road section (tr_l) is then defined as:

$$tr_l = \frac{Q}{(t_c - t_f)L} = \frac{K}{t_f L} \quad (6)$$

Continuance is always positive and directly proportional to the capacity flow of a road section. Continuance captures, for a given section of road, the maintenance of travel time as traffic strain occurs due to lags from road positioning with different densities of uncongested flow. For roads with multiple lanes, the measure of continuance can be taken as the simple summation for each lane - though in practice there will be interference between the lanes.

4.2. The continuance of a vertex

Vertex continuance depends upon the continuance of the intersecting links and the type

of vertex (e.g. roundabout, signal-controlled junction etc.). The orientation of a link (Figure 5) reflects the directionality of traffic flow with respect to a defined co-ordinate system for the whole road network. For example, traffic networks that can transmit traffic in mutually perpendicular directions are better formed and robust than networks transmitting traffic in one direction only.

[Figure 5 near here]

After resolving the flow in two directions and applying co-ordinate transformation (Liu, 2013), the continuance (M_l) of the i^{th} road link can be written into a matrix form:

$$[M_l] = \frac{q}{(t_c - t_f) \cdot L} \cdot \begin{bmatrix} \cos^2 \theta & \sin \theta \cos \theta & 0 & 0 \\ \sin \theta \cos \theta & \sin^2 \theta & 0 & 0 \\ 0 & 0 & \cos^2 \theta & \sin \theta \cos \theta \\ 0 & 0 & \sin \theta \cos \theta & \sin^2 \theta \end{bmatrix} \quad (7)$$

The off-diagonal terms in Equation (7) are null, indicating no coupling between the opposite directions. It is also assumed that the properties are the same in each direction and no delays occur at the vertices. The *vertex continuance matrix* $[M_v]^i$ is taken as the summation of the link continuance matrices for the links starting from the i^{th} vertex i.e.

$$[M_v]^i = \sum_j [M_l]^j \quad j = 1, 2, \dots, N_l^i \quad (8)$$

where $[M_l]^j$ is the link continuance matrix in Equation (8) for link j , N_l^i is the number of links from vertex i .

The continuance (tr_v) of a vertex i contained in a road circuit is defined as:

$$tr_v^i = \det([M_v]^i) \quad (9)$$

Since the determinant of a matrix is equal to the product of its eigenvalues and the eigenvalues of matrix M are orthogonal, Equation (9) captures the continuance characteristics in the two mutually perpendicular directions.

4.3. Well-formedness of a circuit

The well-formedness of a road circuit ($W_{f,cir}$) is defined as the sum of the continuances of vertices contained in that circuit divided by the number of vertices in the circuit as in Equation (10).

$$W_{f,cir} = \frac{\sum_i tr_v^i}{N_{v,cir}} \quad i = 1, 2, \dots, N_{v,cir} \quad (10)$$

where $N_{v,cir}$ is the total number of vertices in the circuit. This measure complies with the criteria stated at the head of Section 4 and includes the qualities of all of the vertices in the road circuit from all of the links including those from outside of the circuit.

In summary this proposed well-formedness measure is based on the form and number of connections of a vertex and the ease of continuation of flow through.

5. Clustering and the hierarchy

The purpose of clustering circuits consisting of links and vertices is to make clear the interconnections within a network at different levels of granularity. The hierarchy of clusters can then be used to search efficiently for ways in which those interconnections may be damaged. Scenarios of sequences of damage that are particularly vulnerable may then be identified.

5.1. A road cluster

A road cluster is a subset of a network such that no vertex in the cluster is disconnected from the rest. A leaf cluster is a single road link, all others (except the entire network) are branch clusters. The circuits inside a cluster are better connected to each other than to any other outside of that cluster. This simply means that vertices inside the cluster can be reached more easily from each other than from vertices outside of the cluster because there are more choices (in terms of travel time, trip length etc) with less costs.

5.2. Criteria for forming a road cluster

Road clusters are grown using the following measures in order of priority. The priority is based on the richness of the measure for robustness and the need to avoid disproportionate consequences.

Well-formedness of a cluster

The well-formedness of a cluster is defined as:

$$W_{f,cl} = \frac{\sum_i tr_v^i}{N_{v,cl}} \quad i = 1, 2, \dots, N_{v,cl} \quad (11)$$

where tr_v^i is the continuance of vertex i , $N_{v,cl}$ is the total number of vertices in the cluster cl .

Algebraic connectivity

This is the second smallest eigenvalue (λ_2) of the Laplacian matrix of a graph (Chung 1997). It depends on the number of vertices in a cluster and mean vertex degree. The larger the algebraic connectivity, the more difficult it is to disconnect the graph and the

more rapidly a damaged network can be returned to a stable state. It is therefore an indicator of the potential for damage and its consequence.

Number of common vertices

The number of common vertices between two road circuits can affect the number of vertices in the Hamiltonian Cycle in the resulting cluster. In fact, if there is only one common vertex between two road circuits there is no Hamiltonian Cycle in the cluster.

Mean distance from the reference nodes

A reference node is the one where traffic originates or sinks. The mean distance from these is calculated as:

$$\Delta_{cl} = \frac{\sum_{i=1}^{N_v} \sum_{j=1}^{N_{rn}} \delta_{i,j}}{N_v} \quad (12)$$

where $\delta_{i,j}$ is the shortest path in terms of length between the i^{th} vertex and the j^{th} reference node; N_{rn} is the number of reference nodes; N_v is the number of vertices in the cluster cl .

The further away a cluster is from a reference node the smaller the potential consequences of damage to that cluster.

Minimum damage demand

Damage demand (Section 6.2) is a measure of the effort needed to damage a link. Minimum damage demand is the smallest damage demand of all the links in the cluster.

5.3. The clustering process

The flowchart of Figure 6 summarises the clustering process. The process begins by identifying, numbering and ranking all of the basic road circuits of a road network according to their well-formedness. Clusters are grown by including the neighbouring circuits that increase (or decrease the least) the highest priority criterion i.e. well-formedness. Where two clusters have the same well-formedness, the second priority criterion is used. Where the second priority criterion fails to discriminate then the third priority criterion is used and so on. Chains of links that connect to only one cluster are included as part of that cluster. Chains formed of single links between clusters are clustered at the same time as its end clusters. Circuits that include one or more reference nodes are not clustered until all other circuits have been clustered and then they are clustered in the same way.

[Figure 6 near here]

5.4. Hierarchical model of road network

A road network R is represented as a hierarchy of clusters R_i^l as holons:

$$R = \{R_i^l | l = 1, 2, \dots, h; i = 1, 2, \dots, n\} \quad (13)$$

where h is the total number of levels in the hierarchy; n is the number of road clusters at level l .

The characteristics of the cluster holons at any particular level emerge through interactions between the cluster holons at lower levels. As the clusters are themselves clustered then the higher levels contain smaller numbers of clusters until at the top level only one cluster remains – i.e. the whole network.

6. Unzipping of clusters and failure scenarios

The computational effort of searching a network for all possible failure scenarios is prohibitive. The hierarchical model facilitates an efficient search process. By working top down through the circuits connecting the clusters at each level of hierarchy a systematic search of the effects of damage is possible. At a given level in the hierarchy candidate clusters and links are damaged and the effects calculated and compared. The connected clusters are then ‘unzipped’. This leads to a set of potential vulnerable failure scenarios.

6.1. Deteriorating event

A deteriorating event is the loss of capacity to transmit traffic along a road link in a network. In the simplest case considered here a deteriorating event removes a link completely. Damage to a node is likely to affect all the incident links. The nature of what causes these events is important but is not the concern of this analysis. However a measure of the effort to cause the damage, called the damage demand, is defined in the next section. A road circuit is either a basic circuit (containing only vertices and links but no clusters) or a connected set of clusters. A deteriorating event within a basic circuit is sufficient to cause the loss of that circuit. A deteriorating event within a cluster results in damage at that level and causes the cluster to separate into its constituent parts. A set of deteriorating events may therefore lead to the total or partial failure of a cluster. A road cluster fails totally when all of its reference nodes are separated from all other reference nodes. A cluster that does not contain any reference nodes may be partially, but not totally damaged when one or more links are damaged.

6.2. Damage demand

Damage demand (a different concept from traffic demand) is a measure of the effort needed to cause a deteriorating event. The actual causes of damage (such as traffic accidents, road repairs or high traffic demand leading to jams) and the efforts involved

can be quite different. However it is plausible to assume that the minimum effort required to cause a deteriorating event is proportional to the properties of the link itself. Damage demand (D) is therefore assumed to be equal to the continuance of the link:

$$D = \frac{Q}{(t_c - t_f) \cdot L} \quad (14)$$

where Q is capacity flow; t_c is the capacity flow travel time; t_f is the free flow travel time; L is the length of the link.

The damage demand of more than one deteriorating event is assumed to be the sum of damage demands of each link. Relative damage demand (D_{rel}) is a non-dimensional measure where damage demand is normalized with respect to the damage demand to cause damage to all the links in the network. It is defined as:

$$D_{rel} = \frac{\sum_{i=1}^n D^i}{\sum_{j=1}^m D^j} \quad (15)$$

where D^i is the damage demand for event i ; n is the number of events; D^j is the damage demand for link j ; m is the number of links in a network.

6.3. Failure consequence

Two of the most important aspects of the consequences of deteriorating events are separateness and loss of function. Separateness is a change in the form of a road network. Loss of function is a change in accessibility between pairs of reference nodes. The severity of these consequences may vary from a ‘little’ (when reference nodes are connected by a reduced number of the paths) to ‘extreme’ when all of the reference nodes are separated.

Separateness (S) is defined as a ratio of the loss in well-formedness to the well-formedness of the intact network:

$$S = \frac{W_f(R) - W_f(R')}{W_f(R)} \quad (16)$$

where $W_f(R)$ is the well-formedness of the intact road network and $W_f(R')$ is the well-formedness of the deteriorated road network. S is a measure with a range $0 \leq S \leq 1$. When $S = 0$, there is no deteriorating event to the network and when $S = 1$, all of the vertices are separated from each other in a network, i.e. no link exists in the network. A negative value may be encountered in some cases, such as when a weakly connected ‘spur’ link is damaged.

A loss of function (F) is defined as:

$$F = \frac{P(R) - P(R')}{P(R)} \quad (17)$$

where $P(R)$ is the number of paths between pairs of reference nodes in the intact network and $P(R')$ is the number of paths between pairs of reference nodes in the deteriorated network. F is a measure with a range $0 \leq F \leq 1$.

A consequence (C) depends on S and F as follows:

$$C = \max(S, F) \quad (18)$$

Thus the range of C is $0 \leq C \leq 1$. There are five possibilities for the values of C as shown in Table 1.

[Table 1 near here]

6.4. Vulnerability index

The vulnerability index (VI) of a failure scenario (Section 6.5) is defined as the ratio of the consequences to the relative damage demand of that scenario i.e.

$$VI = \frac{C}{D_{rel}} \quad (19)$$

VI is a measure of the disproportionateness of the consequences in relation to the damage for a given failure scenario and is not intrinsic to a network. It is non-dimensional so comparisons can be made between failure scenarios in a road network or those between different networks.

6.5. Failure scenarios

A failure scenario is defined as a series of deteriorating events in which some vertices are disconnected from others in a road network. There may be a very large number of failure scenarios since all possible ways in which vertices can be disconnected is enormous. A number of scenarios may be of particular interest to various stakeholders such as the responsible local and regional planning bodies. Four particular types of failure scenarios are:

- (i) minimum failure scenario – with minimum consequence;
- (ii) maximum failure scenario – with maximum vulnerability index;
- (iii) total failure scenario – where one of the consequences is equal to 1 and all of the reference nodes are disconnected from each other;
- (iv) minimum damage demand scenario – the easiest way (the way requiring least effort expressed as damage demand) to cause damage to a network.

6.6. Unzipping algorithm

The unzipping process comprises two parts (a) identifying the cluster to be damaged and (b) identifying the links in that cluster to be damaged. Table 2 sets out the criteria, in order of importance, to select a candidate cluster or link among all the options with corresponding justifications. A cluster is damaged by separating its sub-clusters from each other. Each separation consists of a set of links that are identified in each step of the search process. Together these links form a candidate failure scenario.

[Table 2 near here]

The flowchart for the unzipping process is given in Figure 7. An accompanying suite of programs in MATLAB have been developed to generate the candidate failure scenarios and to calculate the associated vulnerability indices.

[Figure 7 near here]

In order to identify further deteriorating events that will lead to total separation, the damaged network is re-clustered and a new corresponding hierarchical model generated. The next event is identified using the same unzipping process but using the new hierarchy. The process of re-clustering and unzipping after each event is repeated until total separation occurs or all functionality is lost. Theoretically re-clustering and unzipping can be carried out many times to identify all of the candidate scenarios for a network but we will assume that two sets of analyses are sufficient. The first set of candidate scenarios are identified through the unzipping process without re-clustering and the second set of scenarios is generated after the first link in each scenario in the first set is removed and the network is re-clustered. Of course, such vulnerable failure scenarios based on the connectivity of a network have to be seen in the context of traffic on the network.

7. Examples

The analysis of vulnerability will now be illustrated through two examples. The first example is purposely simple in order to explain the formation of the hierarchy. The second example is based on a real transport network.

7.1. Example 1

This is a small network with 10 vertices and 19 links (Figure 8). Vertices x_1 and x_{10} are reference nodes. Each link has one lane in either direction with a capacity of $Q = 2330$ pcu/hour/lane. The free uninterrupted flow speed v_f is 115.8 km/hour, and the capacity speed is 55.6 km/hour. These result in link 4-7 having the lowest continuance and link 7-8 the highest.

[Figure 8 near here]

There are 10 basic circuits. Circuits formed by the vertices 6, 7 and 8, vertices 7, 9 and 10, vertices 7, 8 and 9, and vertices 7, 8 and 10 have the best well-formedness (5061). Circuits formed by the vertices 4, 5, 6 and 7 have the least well-formedness (726).

Circuit 1 (Figure 9) is selected as the seed cluster. This is grown by including the neighbouring Circuit 3 to form Cluster 11 (Table 3). Cluster 11 cannot be grown by including any of the neighbouring circuits because that would result in a decrease in the well-formedness. Hence Circuit 5 is started as a new seed. This is grown by including Circuit 7 to form Cluster 12 and then Circuit 8 to form Cluster 13. Cluster 13 cannot be grown to increase in the well-formedness. Hence Cluster 13 and Cluster 11, which are in series, are merged (Table 3) to form Cluster 14 which leads to a least decrease in the well-formedness. Cluster 14 is grown by including the neighbouring Circuit 10 to form Cluster 15 while the well-formedness is decreasing. Since all of the non-reference circuits (i.e. circuits not containing any reference vertex) are clustered, reference circuits, Circuit 2 and 4 are merged into Cluster 16 at the same time since they have the same well-formedness and other clustering measures. The cluster cannot be grown because it has no neighbouring reference circuits. Another reference circuit, Circuit 6 is selected as a seed to grow to Cluster 17 by including Circuit 9. Clusters 15 and 16 form Cluster 18 of which the well-formedness is decreased least. Cluster 18 is grown by including Cluster 17 to form Cluster 19, which is the whole network.

[Figure 9 near here]

[Table 3 near here]

[Figure 10 near here]

Figure 10 shows the resulting hierarchy of clusters. All the basic circuits, Circuits 1 to 10, are at the lowest level. However for the ease of understanding, some of the circuits are shown near the clusters in which they are included. The well-formedness for each cluster is shown in the hierarchy. The two reference nodes of the network in this example are apart from each other and a reference cluster (called so because it contains a reference node) is formed for each of the two nodes. The two reference clusters are connected by a non-reference cluster. The reference cluster, Cluster 16 is the best formed cluster among the three clusters 15, 16 and 17. Reference node 10 is better connected to its neighbouring vertices and those inside the non-reference cluster.

Each cluster in the hierarchy is unzipped independently. Figure 11 illustrates the unzipping process for Cluster 11 in Figure 10. The search for the first deteriorating event is as follows:

- Start from Cluster 11 with two child clusters, Circuit 1 and Circuit 3. They have the same well-formedness, algebraic connectivity and minimum damage demand. Thus choose Circuit 1 at random (criterion 3c - see Table 2) to be separated from Circuit 1.

- Search down Circuit 1. There are three links in this cluster: links 6-7, 7-8 and 6-8. Link 7-8 is the common link between Circuit 1 and Circuit 3. Thus the other two links are examined. Each of the two links has a common vertex between Circuit 1 and Circuit 3. Both links appear only in Circuit 1. Link 6-8 has the smaller damage demand. Thus link 6-8 is chosen to be damaged (criterion 4f).
- The failure of link 6-8 does not result in either the separation of Circuit 1 and Circuit 3 or a total failure for the network. The search continues.

The search for the second deteriorating event has the following steps:

- There is one link remaining in the original Circuit 1, link 6-7. Select this link to be damaged (criterion 4).
- The failure of link 6-7 results in the separation of Circuit 1 and Circuit 3. The search for this failure scenario stops and the corresponding indices (consequence, relative damage demand and vulnerability index) are calculated.

A selection of results of the unzipping process is presented in Table 4 in three sets: (a) without re-clustering, (b) with re-clustering after the first deteriorating event in (a) and finally (c) with re-clustering after each scenario in (a).

[Table 4 near here]

Some of the scenarios of Table 4 are shown on Figure 8 as dotted lines. The *maximum failure scenario* is b-1 with a VI of 10. The *minimum damage demand scenario* is a-10 with a damage demand of 0.02 and it is also the *minimum failure scenario* with a consequence of 0.03. If link 4–7 is removed in that scenario then there are still several routes between reference nodes 1 and 10 and the consequences to the form of the network are small. However vertex 6 in the damaged network becomes a pressure point and vulnerable since all of the routes connecting the two reference nodes have to pass through it. This manifests itself in scenarios b-1, b-2, b-3 and c-1 where damage to link 4–7 is part of total failure scenarios.

It is important to note that the search for failure scenarios after re-clustering finds new total failure scenarios i.e. scenarios not found in the search without re-clustering. This is a strong argument for the extra computation. On the other hand the scenarios in (c) such as c-1 and c-11 include extra links over those in (b) and consequently have smaller vulnerability indices.

7.2. Example 2

Figure 12 presents a graph model used by Baughan *et al* (2009) and is here used as a road network. The vertices represent cities of the Netherlands and links represent roads connecting the cities. Vertices 23, 42 and 56 are selected as reference nodes since they represent big cities. The free uninterrupted flow speed and the capacity speed are assumed to be the same for all the links; however, the orientation and length of the links are different.

[Figure 12 near here]

[Figure 13 near here]

Following the clustering process given in Figure 6, the clustering sequence is shown in Figure 13. There are 25 basic circuits as labelled. Two circuits, 6 and 16, overlap. Figure 13(a) shows all of the 9 clusters (in different colours/grey shades) in the clustering process when well-formedness is increasing. These 9 clusters are merged into one cluster (Figure 13(b)) and the single links are added to the resulting cluster. The remaining white circuits are the reference circuits to be grouped, one by one, in the remaining stages (Figure 13 (c) to (f)) of the clustering process. Altogether, there are 18 levels in the clustering hierarchy. This hierarchy guides the search for vulnerable failure scenarios.

Again the searching or unzipping process was performed with and without re-clustering after deteriorating events. Some of the candidate failure scenarios with the associated indices are summarised in Table 5 (see Liu, 2013 for a complete list of scenarios).

[Table 5 near here]

In Table 5 the *maximum failure scenario* is Scenario 1 with the highest vulnerability index of 67 but it is a partial and not a total failure scenario. Scenario 10 is a *total failure scenario* and contains Scenario 1. Both are shown in Figure 12 where it is clear that they isolate reference nodes.

The *minimum failure scenario* is Scenario 70 with a consequence of 0.002. Although link 56–59 connects to a reference node (vertex 56) the accessibility between this node and the other two reference nodes (vertices 23 and 42) is little affected.

Scenarios 2 and 3 are non-obvious partial failure scenarios with high VIs of 33.5. Two of the included links are not directly connected to the reference node (vertex 42). The reason for the high VI is that the damage demand of these links is smaller than that of the links connecting directly to the reference node.

[Figure 15 near here]

Partial failure scenario 16, shown in Figure 15, is interesting because it contains the links 35–36, 49–50, 45–47 and 44–57 which are outside of the three reference clusters. If we assume that the three reference nodes (shaded in Figure 15) are big cities and that most city traffic flows within the cities then most journeys are within the reference clusters with smaller flows between them. If the inter-city traffic dominates then one has to pay more attention to this failure scenario.

Failure scenarios containing single links, such as 73, 74, 76, 78 and 79, have low

vulnerability indices because they are connected to vertices with a small vertex degree and are at the boundaries of the network.

8. Conclusions

- (1) The purpose of the analysis of the vulnerability of road traffic networks, presented here, is to identify high consequence failure scenarios that may arise from vulnerable weaknesses in the form of the network which are independent of models of traffic demand or the sources of the damage.
- (2) The theory has been developed through an analogy with structural vulnerability theory using systems thinking. A new measure of continuance is introduced as a basis for the development of a measure of well-formedness. It captures, for a given section of road, the continuation or duration of travel time with different degrees of saturation of uncongested traffic flow as traffic strain occurs due to lags from free uninterrupted flow.
- (3) A hierarchical model representing a road network at various levels of granularity as interconnected cluster holons has been formulated. The building of the hierarchical model begins by identifying basic road circuits. These are then clustered using five nested criteria of well-formedness, algebraic connectivity, number of common vertices, mean distance from a reference node and minimum damage demand.
- (4) An algorithm for searching a hierarchical model of a road traffic network has been presented. The overall purpose of the search is to identify the ways a road network can become partially or completely dysfunctional and in particular to identify high consequence failure scenarios which are independent of models of traffic demand or the sources of the damage.
- (5) A failure scenario is defined as a series of deteriorating events in which some vertices are disconnected from others. The important consequences of deteriorating events have been characterised as separateness and loss of function. Separateness is a change in the form of a road network. Loss of function is a change in accessibility between pairs of reference nodes.
- (6) The importance of the proposed search process is that the computational effort of working through a complete network for all of the possible failure scenarios is prohibitive. By working top down through the circuits of clusters at each level of granularity the new unzipping search of the effects of damage enables a systematic search.
- (7) The clustering and unzipping analysis of the two example networks demonstrates that high vulnerability failure scenarios for a road network are not always obvious. The analysis enables an estimate of consequences but a separate risk analysis is required to identify the likelihood or probability of damage.
- (8) The theory as developed so far applies only to uncongested traffic flow. The purpose of the analysis is not to provide definitive answers of the most vulnerable scenarios. Rather it is to provide an analytical means of exploring the effects of damage on a complex road network so that design, monitoring and maintenance decisions can be made to increase robustness by reducing the vulnerability to damage.

Acknowledgements

The first author appreciates the financial support from the China Scholarship Council Grant.

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